Marginalized Particle Filters for Bayesian Estimation of Gaussian Noise Parameters

Emre Özkan, Saikat Saha and Fredrik Gustafsson

AUTOMATIC CONTROL
REGLERTEKNIK
LINKÖPINGS UNIVERSITET

Bayesian Framework

- Noise Parameters Estimation
- Conjugate Priors
- Marginalization

Introduction

- We propose an efficient method in a Bayesian framework for approximating the joint density of the unknown parameters and the state based on the particle filters and marginalization concepts.
- We assume suitable prior distributions for the unknown noise parameters. Conditional on the particle filter output for the state, we define analytical posterior distribution for the unknown noise parameters and propagate the hyper-parameters of the posterior recursively.

Problem Formulation

$$x_t = f_t(x_{t-1}) + v_t$$
$$y_t = h_t(x_t) + w_t$$

$$v_t \overset{i.i.d.}{\sim} \mathcal{N}(\mu_v, \Sigma_v),$$
$$w_t \overset{i.i.d.}{\sim} \mathcal{N}(\mu_w, \Sigma_w).$$

$$\theta \triangleq [\theta^v, \theta^w] \triangleq [\mu_v \Sigma_v, \mu_w \Sigma_w].$$

The typical problem here is to infer sequentially the unobserved state x_t together with the unknown noise statistics θ based on a set of observation $y_{0:t}$.

Normal-inverse-Wishart Priors

Assuming Normal-inverse-Wishart distribution with hyper-parameters, $(k_0, \mu_0, v_0, \Lambda_0)$ defines a hierarchical Bayesian model given below:

$$[\mu, \Sigma] \sim \mathrm{NiW}(k_0, \mu_0, v_0, \Lambda_0),$$
$$z \sim \mathcal{N}(\mu, \Sigma)$$
$$\mu|\Sigma \sim \mathcal{N}(\mu_0, \frac{\Sigma}{k_0})$$
$$\Sigma \sim \mathrm{iW}(v_0, \Lambda_0)$$

where iW(.) denotes Inverse Wishart distribution. The posterior is again NiW with updated hyper-parameters. Pseudo measurements are used for the hyper-parameters update.

$$z_t^v \triangleq x_t^{(i)} - f_t(x_{t-1}^{(i)}) \quad \text{for the process noise,}$$
$$z_t^w \triangleq y_t - h_t(x_t^{(i)}) \quad \text{for the measurement noise .}$$

Marginalization

The joint distribution of the states and the unknown parameters can be decomposed into conditional distributions:

$$p(x_{0:t}, \theta | y_{0:t}) = p(\theta | x_{0:t}, y_{0:t}) p(x_{0:t} | y_{0:t}).$$

Suppose we approximate the distribution $p(x_{0:t}|y_{0:t})$ by a set of N particles and their weights as

$$p(x_{0:t}|y_{0:t}) \simeq \sum_{i=1}^{N} \omega_t^{(i)} \delta_{x_{0:t}^{(i)}}(.).$$

For each particle we can compute analytical expressions for the posterior distribution of the unknown parameters.

Moreover, the unknown parameters can be integrated out in particle filter update equations.

In likelihood computation and state prediction equations, it is possible to integrate out the unknown noise parameters as they follow normal-inverse-Wishart distribution.

$$p(y_t|x_t) = \int p(y_t|\theta^v, x_{0:t}) p(\theta^v|x_{0:t}) \, d\theta^v.$$

$$p(x_t|x_{0:t-1}) = \int p(x_t|\theta^w, x_{0:t-1}) p(\theta^w|x_{0:t-1}) \, d\theta^w.$$

The resulting predictive distributions are multivariate Student-t distributions.

Simulation Results

We use the following benchmark scalar nonlinear time series model for illustration:

$$x_t = \frac{x_{t-1}}{2} + \frac{25 x_{t-1}}{1 + x_{t-1}^2} + 8\cos(1.2t) + v_t, \tag{1}$$

$$y_t = \frac{x_t^2}{20} + w_t, \quad v_t \perp w_t, \; t = 1, 2, \ldots \tag{2}$$

where $v_t \sim N(\mu_v, \Sigma_v)$ and $w_t \sim N(\mu_w, \Sigma_w)$. In the figure below the estimates for the measurement and the process noise variances and the means are depicted together.

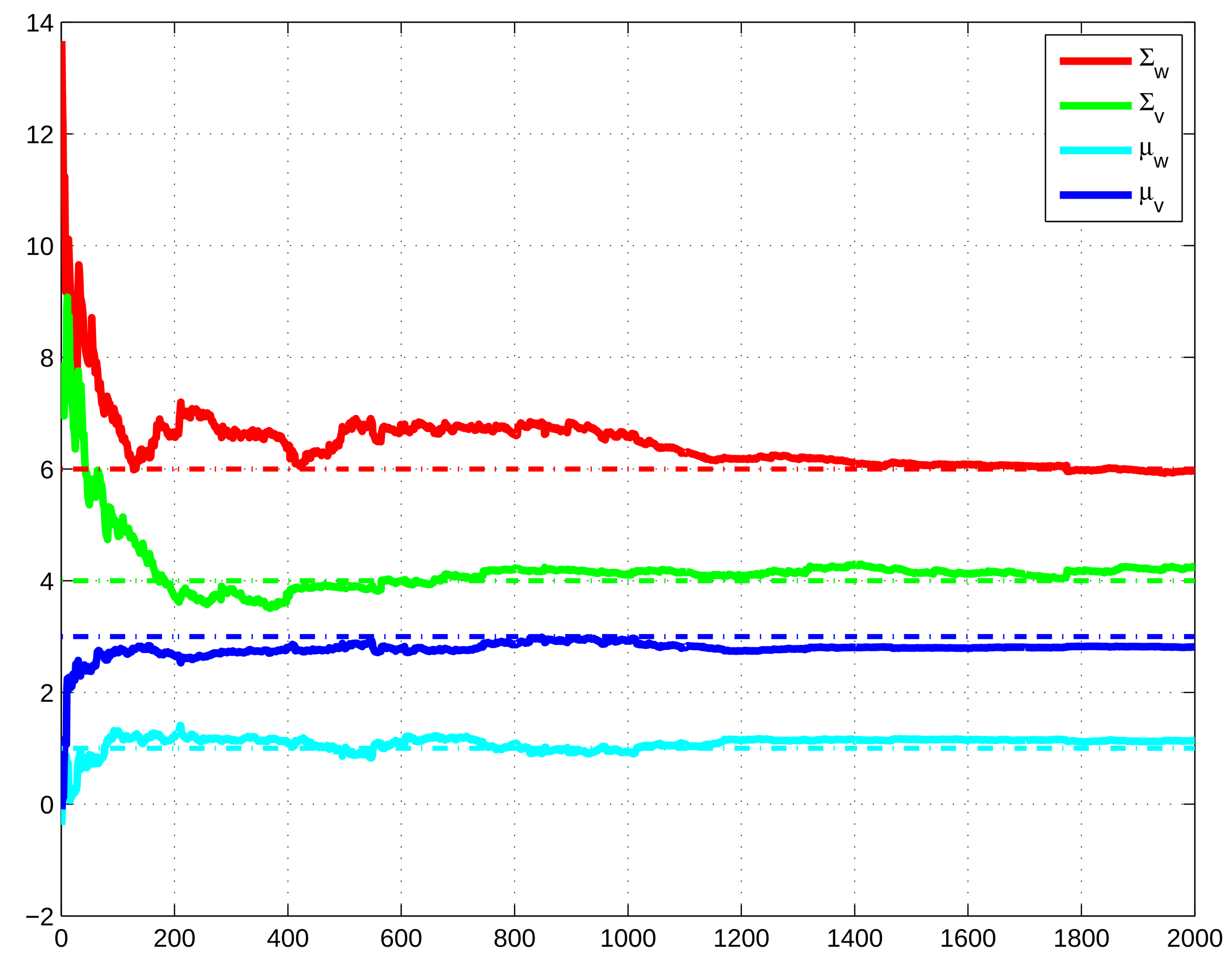

Figure 1: Estimated mean and covariance for the measurement and the process noises. The algorithm is run with 5000 particles.

Future Work

- Extension to time-varying noise parameters
- Larger class of noises with conjugate priors

References

[1] S. Saha, E. Özkan, F. Gustafsson, and V. Šmídl, "Marginalized particle filters for Bayesian estimation of Gaussian noise parameters," Accepted to the 13th International Conference on Information Fusion, 2010.