Horizon Length and Fuel Equivalents for Fuel-optimal Look-ahead Control

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Summary: The two main issues in RHC are addressed for the look-ahead problem. The support for the residual cost used is strengthened by an interpretation in terms of Lagrange multipliers that confirms the physical intuition. Measures for suboptimality are introduced that enables choosing horizon length with the appropriate compromise between fuel consumption and trip time.

Minimum-fuel operation

The objective of minimizing the fuel mass M, with the trip time constraint $T \leq T_0$ on a trip $s \in [0, S]$, is solved by applying RHC with horizon R on

$$J = \min_{u \in U} \left\{ \phi(x(R)) + M(x,u) + \beta T(x) \right\}$$

Fuel equivalents

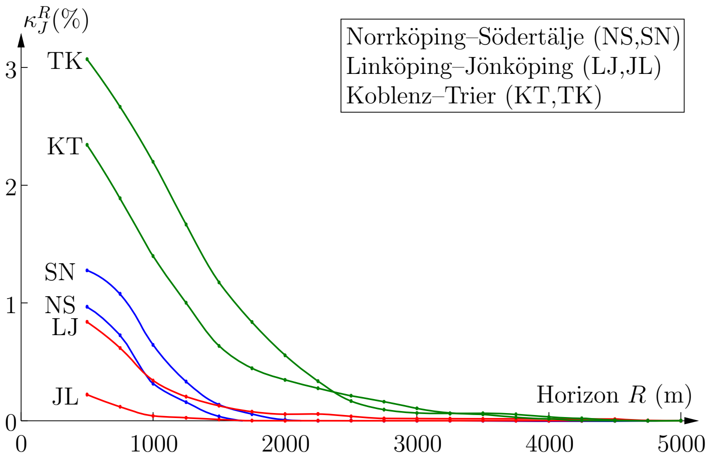
The Lagrange multiplier for the kinetic energy e is shown to be $-\gamma c_m$ on a singular arc, where γ (g/J) is derived from engine and driveline characteristics and $c_m \approx 1$. Using this value as an approximation for constrained arcs as well, the cost function is

$$J(s,x) \approx -\gamma c_m e + C(s)$$

that yields the residual cost $\phi(x) = -\gamma c_m e$.

The factor β corresponds to the constant Lagrange multiplier for time and it is shown that it determines the velocity $\hat{v}$ on singular arcs and that

$$\beta = 2\gamma P_{\text{air}}(\hat{v}) \approx 2\gamma P_{\text{air}}(S/T_0)$$



Suboptimality measures

Define the degree of suboptimality for a horizon length R by κ_J^R,

$$\kappa_J^R = J^S_{\mu^R}(x)/J^S(x) - 1$$

where $J^S_{\mu^R}$ is the cost with the RHC controller μ^R and the optimal cost is $J^S(x) = M^S(x) + \beta T^S(x)$. To separate the suboptimality in fuel mass M and trip time T, define the measures κ_M^R, κ_T^R by

$$\kappa_M^R = M^S_{\mu^R}(x)/M^S(x) - 1, \quad \kappa_T^R = T^S_{\mu^R}(x)/T^S(x) - 1$$

It is shown that:

⋆ The relationship

$$0 \leq \kappa_J^R(1+q) = q\kappa_M^R + \kappa_T^R$$

holds and the solution approaches $q\kappa_M^R + \kappa_T^R = 0$ when $R \to S$.

⋆ A variation in β causes the solution to move along

$$q\kappa_M^\beta + \kappa_T^\beta = 0$$

where q is the compromise between fuel and time,

$$q = \frac{M^S}{\beta T^S} \approx \frac{1}{2}\left(1 + \frac{F_{\text{roll}}}{F_{\text{air}}(\hat{v})}\right)$$

The conclusion is that when choosing the horizon length, with a desired suboptimality d in M, aim for

$$0 \leq q\kappa_M^R + \kappa_T^R \leq d$$