A Robust Particle Filter and a Basic Convergence Result

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Motivation & Results

- Particle filtering methods have become perhaps the most common method for solving nonlinear filtering and sensor fusion problems.
- We present a convergence proof applicable to *unbounded* functions, such as the state estimate itself $\mathrm{E}(x_t|y_{1:t})$. Previous results are only applicable to bounded function.
- The mathematics of the proof require us to modify the algorithm in a sense that can be interpreted as a *robustification*.

Problem Formulation

We are concerned with recursive estimation in nonlinear dynamic systems,

$$x_{t+1} = f(x_t, u_t, t) + v_t,$$
$$y_t = h(x_t, t) + e_t.$$

The state estimate from $\{u_s, y_s\}_{s=1}^{t}$ is

$$\hat{x}_t = \mathrm{E}(x_t|y_{1:t}),$$

and it can be approximated using the filtering density $p(x_t|y_{1:t})$ provided by the particle filter.

Main Result

If $\hat{x}_t^N$ denotes the particle filter estimate we have

$$\hat{x}_t^N \to \hat{x}_t, \quad \text{as} \quad N \to \infty.$$

Technical Formulation

Let $\rho(y_t|x_t)$ denote the likelihood function and let $K(x_{t+1}|x_t)$ denote a Markov transition kernel. Finally, let $\pi_{t|t}$ be the marginal distribution that we seek to estimate.

Hypotheses:

H1. $(\pi_{s|s-1}, \rho) > 0$, $\rho(y_s|x_s) < \infty$ and $K(x_s|x_{s-1}) < \infty$ for given $y_{1:s}$, $s = 1, \dots, t$.

H2. The function $\phi(\cdot)$ satisfies

$$\sup_{x_s} |\phi(x_s)|^4 \rho(y_s|x_s) < C(y_{1:s})$$

for given $y_{1:s}$, $s = 1, \dots, t$.

If H1 and H2 hold, then

$$\mathrm{E}\left|(\pi_{t|t}^N, \phi) - (\pi_{t|t}, \phi)\right|^4 \le C_{t|t}\frac{\|\phi\|_{t,4}^4}{N^2},$$

where $\pi_{s|s}^N$ is generated by the robust PF and

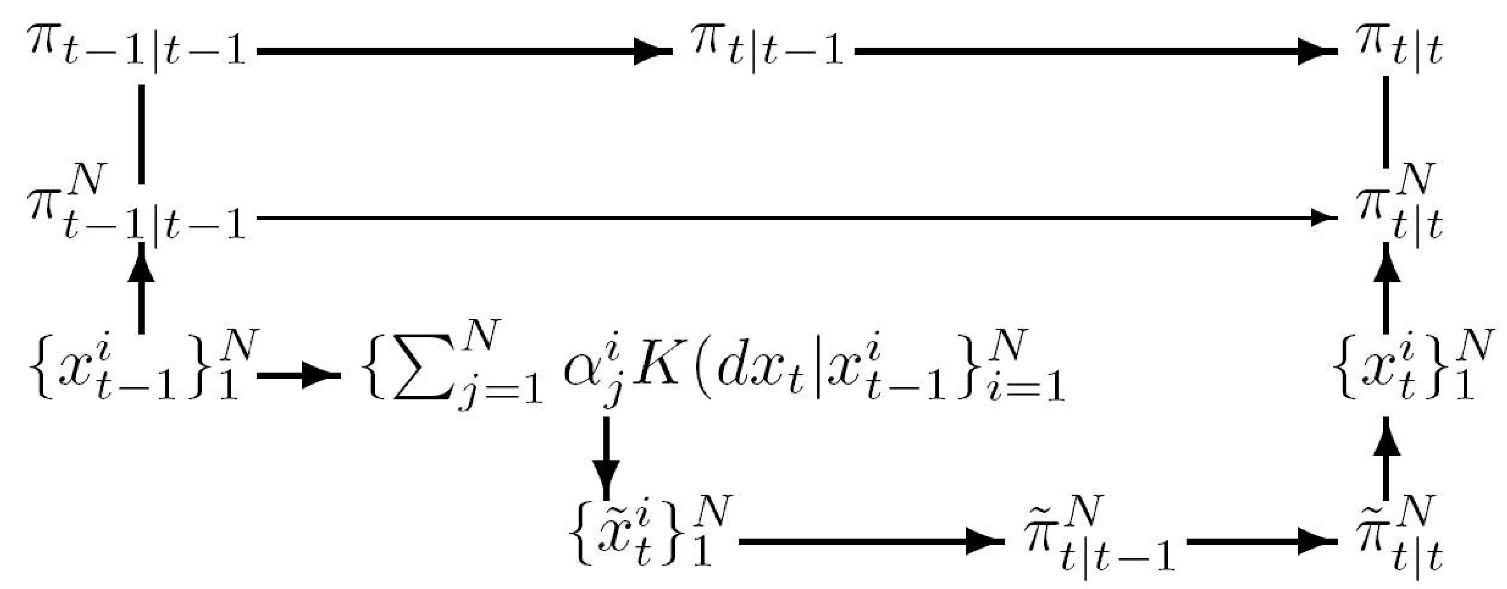
$$\|\phi\|_{t,4} \triangleq \max\left\{1,\ (\pi_{s|s}, |\phi|^4)^{1/4},\ s = 0, \dots, t\right\}.$$

A Robust Particle Filter

In order for the convergence proof to hold we have to impose the (natural) requirement that

$$(\tilde{\pi}_{t|t-1}^N, \rho) = \sum_{i=1}^{N} \rho(y_t|\tilde{x}_t^i) \ge \gamma_t > 0.$$

Here is a brief algorithm description



We can prove that the effect of the modification disappears as the number of particles increase,

$$P\Big((\tilde{\pi}_{t|t-1}^N, \rho) \ge \gamma_t\Big) \xrightarrow[N\to\infty]{} 1,$$

and we can illustrate it numerically as well,