Nonlinear System Identification using EM

Thomas B. Schön, Adrian Wills and Brett Ninness

AUTOMATIC CONTROL
REGLERTEKNIK
LINKÖPINGS UNIVERSITET

Contribution

Presents a **new algorithm** for nonlinear system identification based on Expectation Maximization (EM) and a particle smoother.

Problem Formulation

We consider the problem of identifying the parameters θ in the **nonlinear, non-Gaussian** model

$$x_{t+1} = f_t(x_t, u_t, \theta) + g_t(x_t, u_t, \theta)v_t, \quad (1a)$$

$$y_t = h_t(x_t, u_t, \theta) + e_t. \quad (1b)$$

The parameters are identified using the **maximum likelihood** (ML) method. The estimate

$$\widehat{\theta} = \arg\max_{\theta} p_\theta(y_t, \dots, y_N)$$

is computed using the EM algorithm.

EM Algorithm

The EM algorithm computes the **maximum likelihood** estimates of unknown parameters in probabilistic models involving latent variables.

We have to compute

$$Q(\theta, \theta_k) = I_{\text{prior}} + I_{\text{pred}} + I_{\text{lh}}$$

$$I_{\text{prior}} = \int \log p_\theta(x_1) p_{\theta_k}(x_1 | Y_N) dx_1$$

$$I_{\text{pred}} = \sum_{t=1}^{N-1} \int\int \log p_\theta(x_{t+1}|x_t) p_{\theta_k}(x_{t:t+1}|Y_N) dx_{t:t+1}$$

$$I_{\text{lh}} = \sum_{t=1}^{N} \int \log p_\theta(y_t|x_t) p_{\theta_k}(x_t|Y_N) dx_t$$

(E Step): Calculate

$$Q(\theta, \theta_k) = \mathrm{E}_{\theta_k}\{\log p_\theta(X_N, Y_N) \mid Y_N\}$$

(M Step): Solve

$$\theta_{k+1} = \arg\max_{\theta} Q(\theta, \theta_k)$$

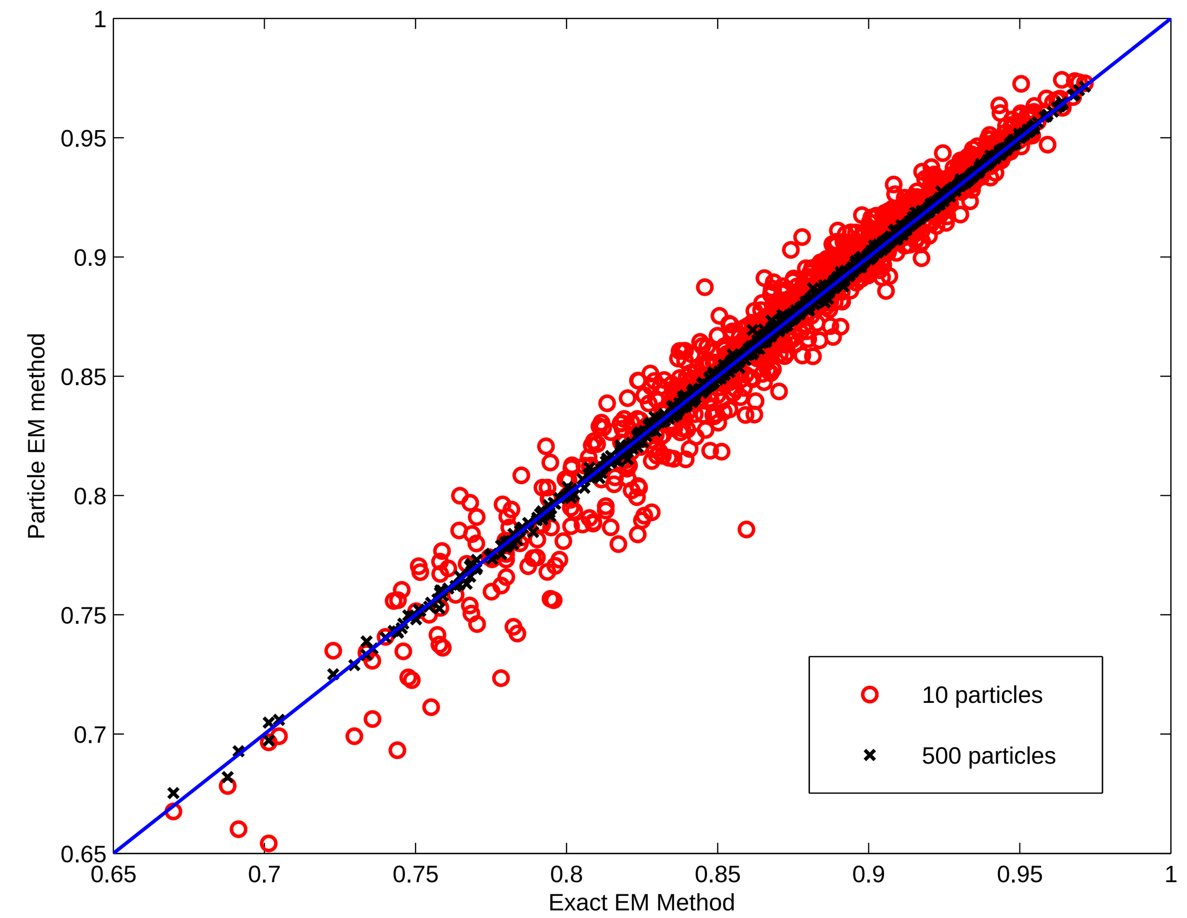
The above integrals lack analytical solutions. Hence, we are forced to **approximations**. Using a **particle smoother** we can approximate $p_{\theta_k}(x_t|Y_N)$ and $p_{\theta_k}(x_{t:t+1}|Y_N)$ arbitrary well,

$$p_{\theta_k}(x_t|Y_N) \approx \sum_{i=1}^{M} w_t^i \delta(x_t - \tilde{x}_t^i)$$

The maximization is solved using **Quasi-Newton** algorithms (BFGS).

Numerical Illustrations

Consider a scalar **linear** state-space model (exact solution is available). Below is a comparison of the exact solution and our algorithm,



The cost function of a more challenging **nonlinear**, non-Gaussian problem with iterates overlayed is given below