

## Extended targets

**Definition:** Extended targets potentially produce more than one measurement per time step.

**Challenge:** Multiple extended targets are usually hard to track, due to the resulting complex data association.

**Solution:** An implementation of a Gaussian mixture Probability Hypothesis Density (PHD) filter for tracking of multiple extended targets.

## The PHD filter

A Poisson random finite set (RFS) is a set with a random number of stochastic states variables

$$\mathbf{X}_k = \{ \mathbf{x}_k^{(1)}, \dots, \mathbf{x}_k^{(N_{x,k})} \}.$$

It is completely described by its PHD  $v(\mathbf{x})$ , and its cardinality distribution,  $\Pr\{|\mathbf{X}| = N_x\}$ . The PHD is here represented by a Gaussian mixture

$$v_k(\mathbf{x}) = \sum_{i=1}^{J_k} w_k^{(i)} \mathcal{N}(\mathbf{x} | m_k^{(i)}, P_k^{(i)}).$$

The number of estimated targets in  $\mathbf{X}$  within the volume  $V$  is given by

$$N_V = \int_V v_k(\mathbf{x}) d\mathbf{x}$$

## Measurement Update

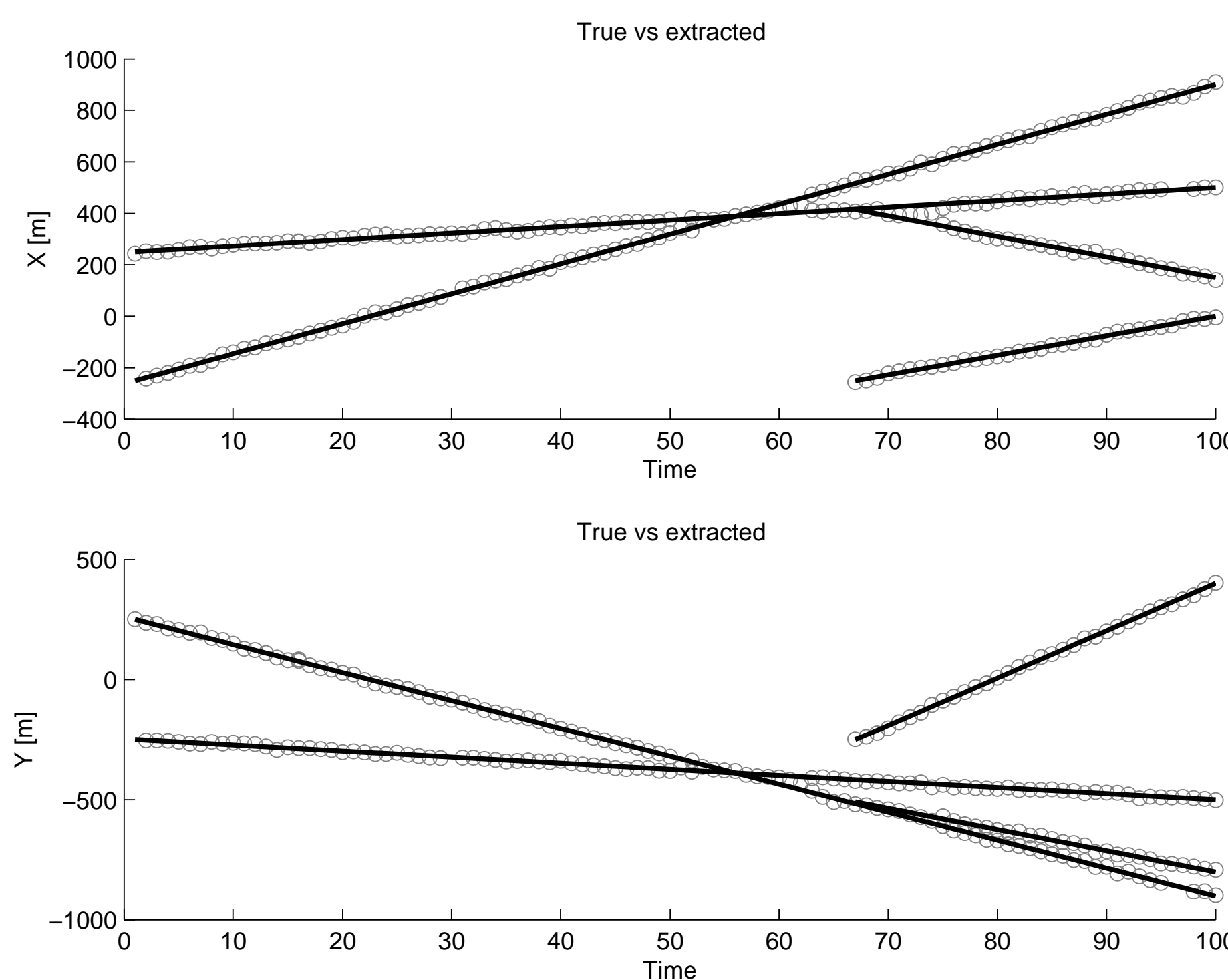
If  $v_{k|k-1}(\mathbf{x}|\mathbf{Z})$  is the predicted PHD-intensity, the corrected PHD-intensity is

$$v_{k|k}(\mathbf{x}|\mathbf{Z}) = L_{\mathbf{Z}_k}(\mathbf{x}) v_{k|k-1}(\mathbf{x}|\mathbf{Z}),$$

where the measurement pseudo-likelihood function is given by

$$L_{\mathbf{Z}_k}(\mathbf{x}) = 1 - \left(1 - e^{-\gamma(\mathbf{x})}\right) p_D(\mathbf{x}) + e^{-\gamma(\mathbf{x})} p_D(\mathbf{x}) \sum_{p \perp \mathbf{Z}_k} \omega_p \sum_{W \in p} \frac{\gamma(\mathbf{x})^{|W|}}{d_W} \cdot \prod_{\mathbf{z} \in W} \frac{\phi_{\mathbf{z}}(\mathbf{x})}{\lambda_k c_k(\mathbf{z})}.$$

## Tracking Simulation



— true location, o estimated location.

## Partitioning the Measurement Set

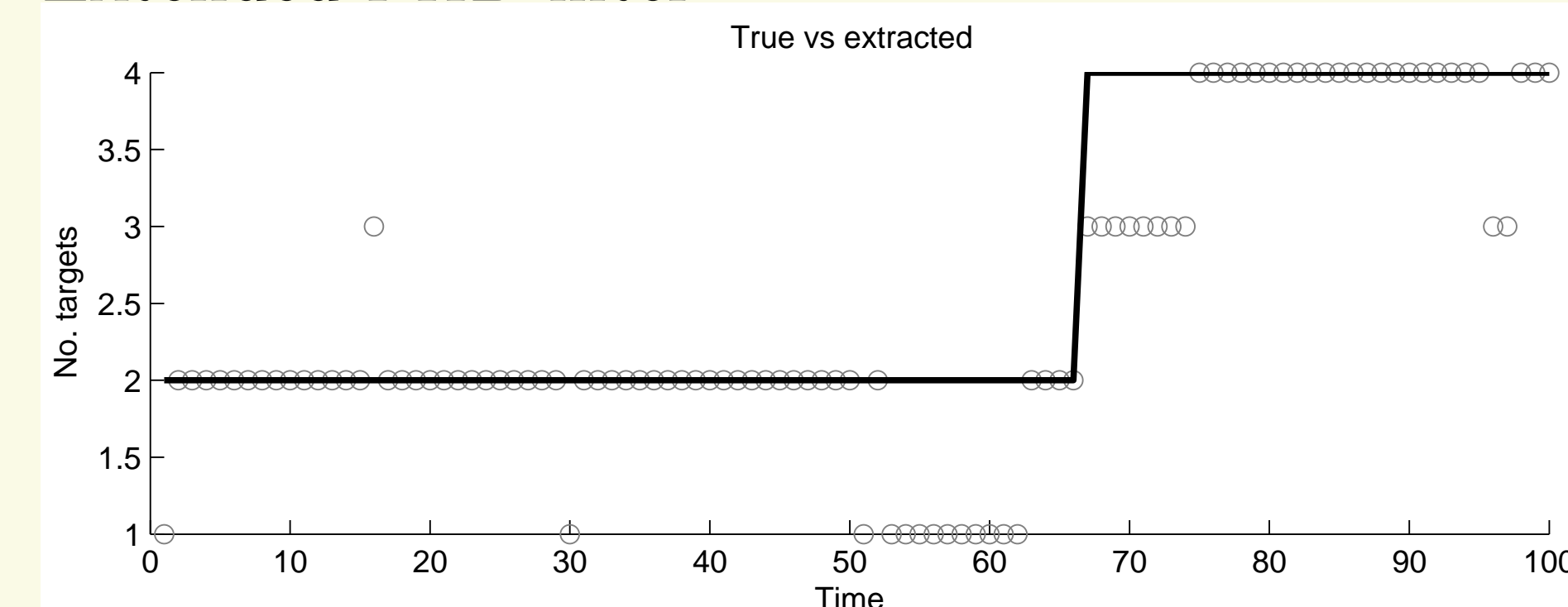
A partition  $p$  is a division of the set of measurements  $\mathbf{Z}$  into subsets, called cells  $W$ . The test for whether  $\mathbf{z}_k^{(1)}$  and  $\mathbf{z}_k^{(2)}$  belong to the same cell  $W$  becomes

$$\left\| \mathbf{z}_k^{(1)} - \mathbf{z}_k^{(2)} \right\|_2 < \sigma_e \sqrt{\delta_{P_G}}.$$

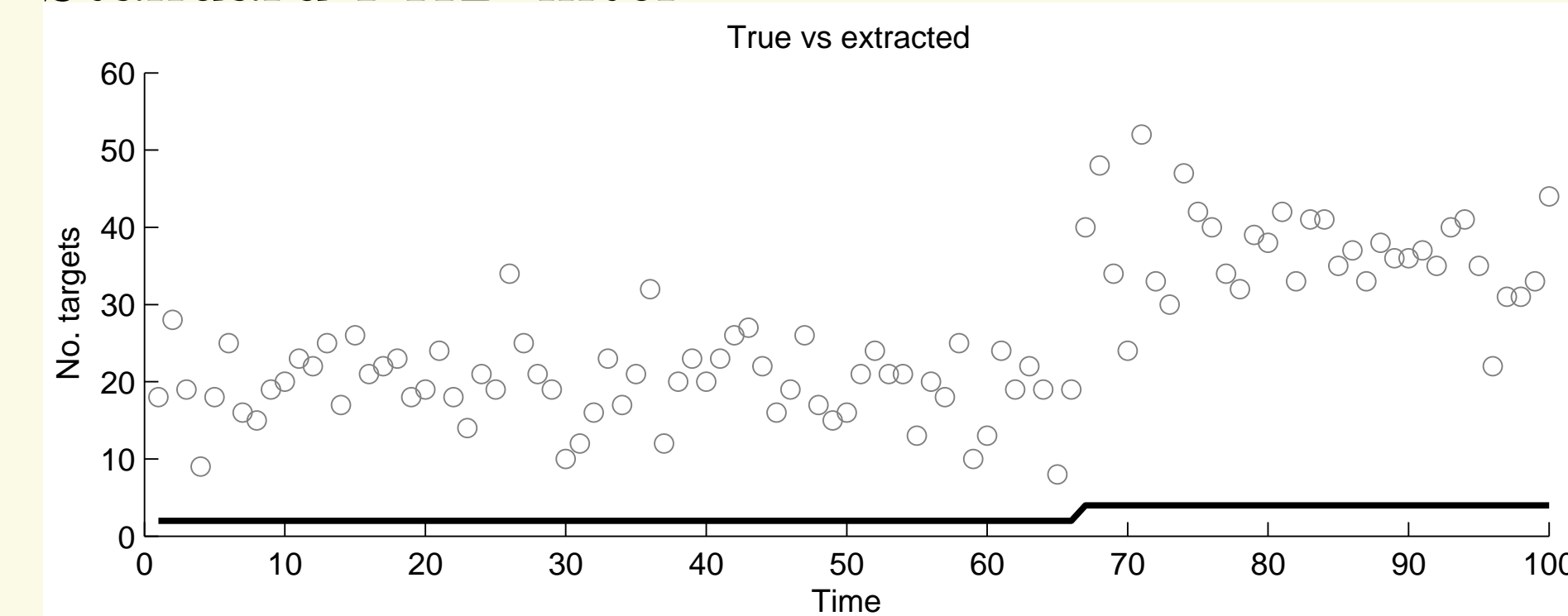
for a unitless distance threshold  $\delta_{P_G}$ .

## Number of Targets Present

### Extended PHD filter



### Standard PHD filter



— true, o estimated.