Isolability Analysis under Causality Constraints

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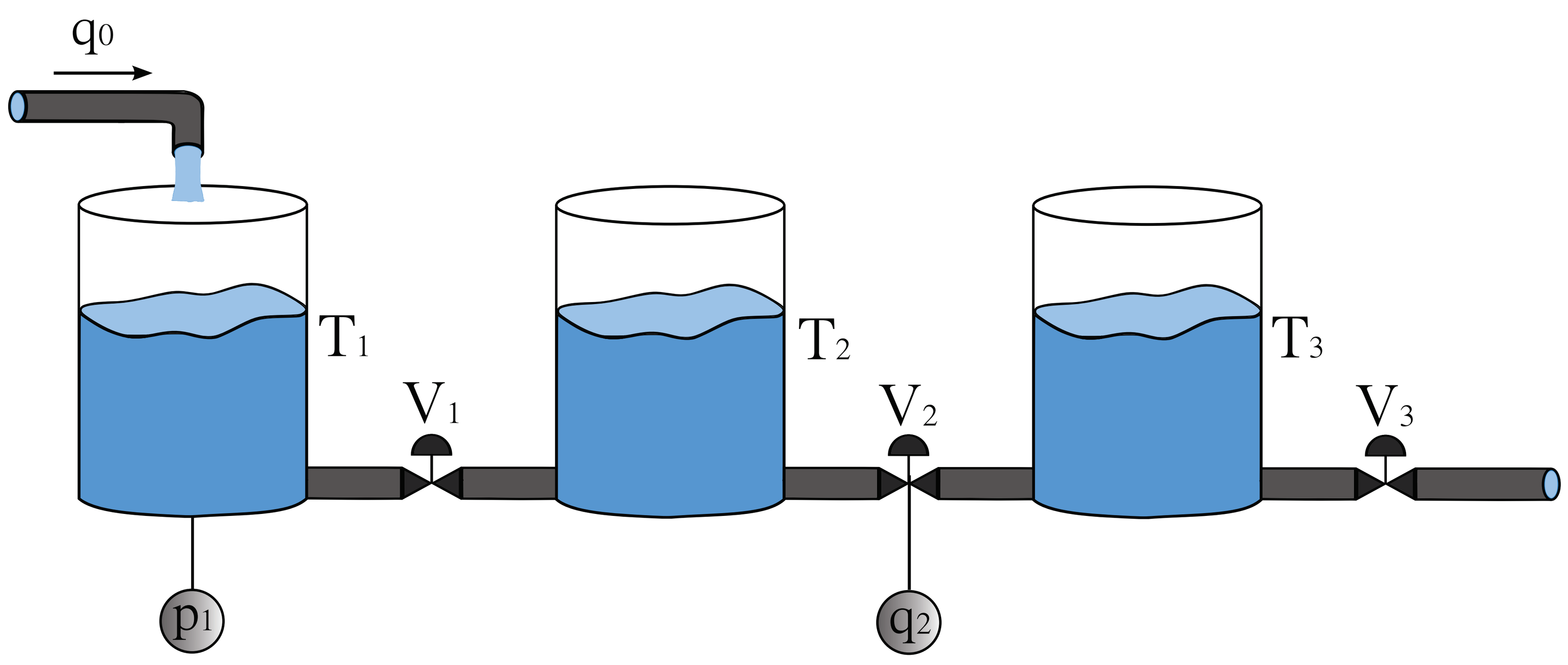
Summary

Sequential residual generation under causality constraints is a popular approach to design residual generators. An objective here is to develop theory and algorithms to determine diagnosability properties under different causality constraints. The algorithms also provide causal matchings that can automatically be transformed into residual generators.

Illustrative example

A three tank system will be used to illustrate the results.



The system can be described by the equations

$$e_1: q_1 = \frac{1}{R_{V1}}(p_1 - p_2) \quad e_2: q_2 = \frac{1}{R_{V2}}(p_2 - p_3) \quad e_3: q_3 = \frac{1}{R_{V3}}(p_3)$$
$$e_4: \dot{p}_1 = \frac{1}{C_1}(q_0 - q_1) \quad e_5: \dot{p}_2 = \frac{1}{C_2}(q_1 - q_2) \quad e_6: \dot{p}_3 = \frac{1}{C_3}(q_2 - q_3)$$
$$e_7: y_1 = p_1 \quad e_8: y_2 = q_2 \quad e_9: y_3 = q_0$$
$$e_{10}: \dot{p}_1 = \frac{d}{dt}p_1 \quad e_{11}: \dot{p}_2 = \frac{d}{dt}p_2 \quad e_{12}: \dot{p}_3 = \frac{d}{dt}p_3$$

Causality Constraints

Integral causality means that only integration of ordinary differential equations and solving algebraic systems of equations are allowed when computing a residual. For example it is possible to use equations e_1, e_4, e_5, e_8, e_9 to compute p_1, p_2, q_0, q_1, and q_2:

$$\dot{p}_1 = \frac{1}{C_1}\left(y_3 - \frac{1}{R_{V1}}(p_1 - p_2)\right), \quad \dot{p}_2 = \frac{1}{C_2}\left(\frac{1}{R_{V1}}(p_1 - p_2) - y_2\right)$$
$$q_0 = y_3, \quad q_1 = \frac{1}{R_{V1}}(p_1 - p_2), \quad q_2 = y_2$$

Now e_7 can be used as redundant equation to create the residual $r = y_1 - p_1$ where p_1 is the computed value and y_1 is the measured value.

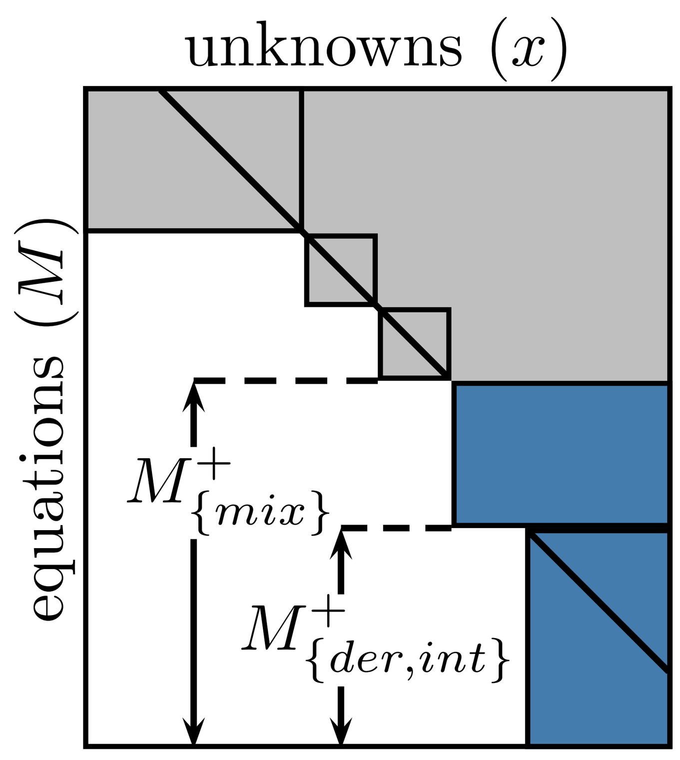
In the same way derivative causality means that differentiation and solving algebraic systems of equations are allowed. For example, e_1, e_4, e_7, e_8, and e_9 can be used to compute p_1, p_2, q_0, q_1, and q_2

$$p_1 = y_1, \quad q_0 = y_3, \quad q_2 = y_2, \quad \dot{p}_1 = \frac{d}{dt}p_1$$
$$q_1 = q_0 - C_1\dot{p}_1, \quad p_2 = p_1 - R_{V1}q_1$$

Mixed causality means that both differentiation and integration are allowed.

Isolability and Causal Matchings

An objective is to compute diagnosability properties of a given model under certain causality constraints. The key tool is the decomposition of the model structure shown in the figure. The blue areas indicate the parts of the model that contain redundancy under the causality constraint, i.e. the equations in M that can be monitored.



Definition 1 (Detectability). *A fault f, with fault equation e_f, is detectable under a causality constraint $c \in \{mix, der, int\}$ if $e_f \in M_c^+$.*

Definition 2 (Isolability). *A fault f_1 is isolable from a fault f_2, under a causality constraint $c \in \{mix, der, int\}$, if $e_{f_1} \in (M \setminus \{e_{f_2}\})_c^+$.*

Efficient algorithms to compute M^+_{mix}, M^+_{der}, and M^+_{int} have been developed that also deliver a causal matching.

Isolability for the Example

To illustrate single fault isolability properties of a model, an isolability matrix can be computed where an X in position (i, j) indicates that fault j can not be isolated from fault i. For derivative and integral causality we obtain:

Derivative causality

	R_{V_1}	R_{V_2}	R_{V_3}	C_{T_1}	C_{T_2}	C_{T_3}
R_{V_1}	X	X	X	X	X	X
R_{V_2}		X	X			X
R_{V_3}		X	X			X
C_{T_1}	X	X	X	X	X	X
C_{T_2}					X	
C_{T_3}		X	X			X

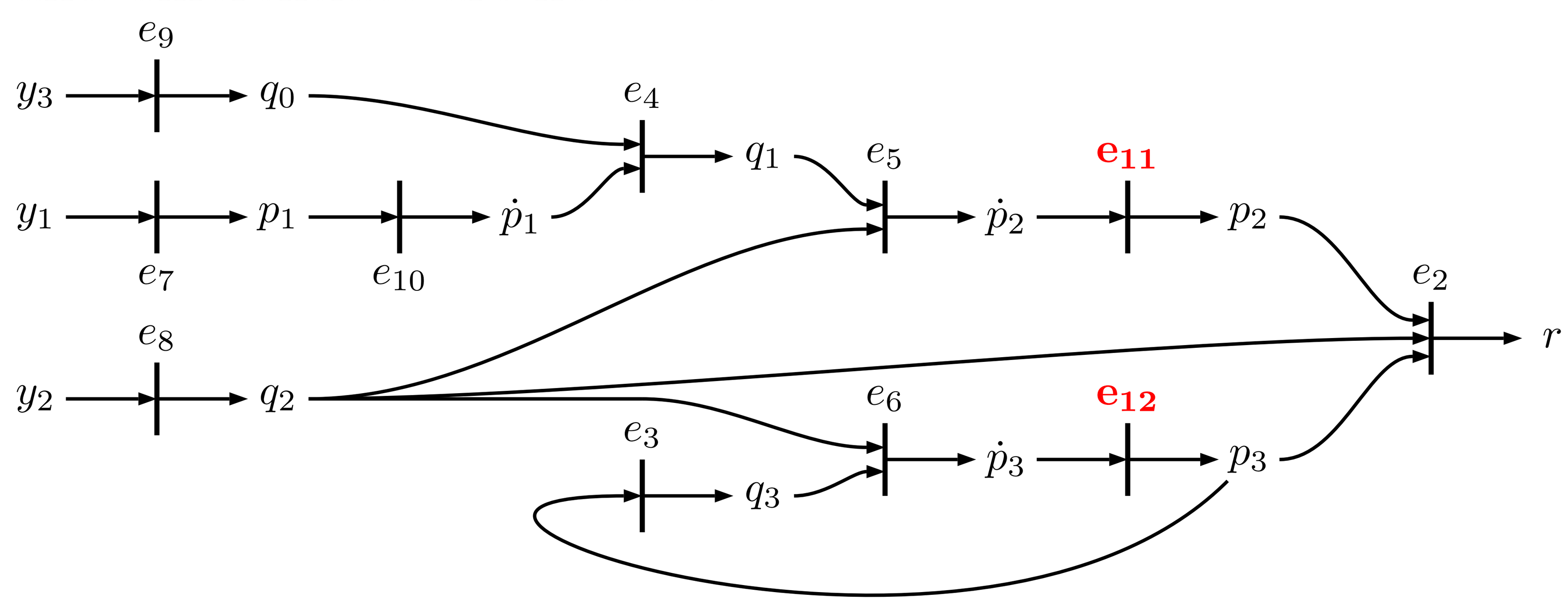
Integral causality

	R_{V_1}	R_{V_2}	R_{V_3}	C_{T_1}	C_{T_2}	C_{T_3}
R_{V_1}	X	X	X	X	X	X
R_{V_2}		X	X			X
R_{V_3}		X	X			X
C_{T_1}				X		
C_{T_2}					X	
C_{T_3}		X	X			X

and with no causality constraint according to the table on the right. Thus, here we have $\mathcal{I}_{der} \prec \mathcal{I}_{int} \prec \mathcal{I}_{mix}$ but in general, only $\mathcal{I}_{der} \prec \mathcal{I}_{mix}$ and $\mathcal{I}_{int} \prec \mathcal{I}_{mix}$ can be guaranteed. Full isolability can not be reached in this example with derivative or integral causality constraints since residuals where you need to both integrate and differentiate are needed:

Mixed causality

	R_{V_1}	R_{V_2}	R_{V_3}	C_{T_1}	C_{T_2}	C_{T_3}
R_{V_1}	X					
R_{V_2}		X	X			X
R_{V_3}		X	X			X
C_{T_1}				X		
C_{T_2}					X	
C_{T_3}		X	X			X



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