Fault Diagnosis of Models With Constraints

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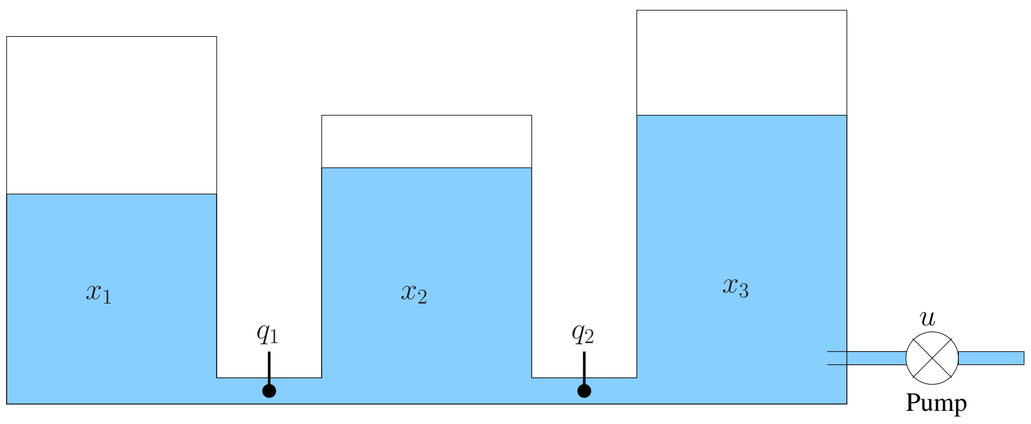
Summary

Structural analysis is a standard tool to identify submodels that can be used to design model based diagnostic tests. Structural approaches typically operate on models described by a set of equations. This work extends such methods to be able to handle models with constraints, e.g. inequality constraints on state variables.

Three Tank System

The following water tank system will be used to illustrate the basic concepts



There are two sensors measuring the level x_1 and the flow q_1. Consider the following model of the system:

$$e_1 : \dot{x}_1 = -q_1, \quad e_4 : q_1 = x_1 - x_2, \quad e_6 : y_1 = x_1$$
$$e_2 : \dot{x}_2 = q_1 - q_2, \quad e_5 : q_2 = x_2 - x_3, \quad e_7 : y_2 = q_1$$
$$e_3 : \dot{x}_3 = q_2 - u$$

Four tests can be found using conventional structural methods:

$$\dot{y}_1 + y_2 = 0, \quad \ddot{y}_1 + 2\dot{y}_1 - \ddot{y}_2 - 3\dot{y}_2 - y_2 + u = 0$$
$$y_1^{(3)} + 4\ddot{y}_1 + 3\dot{y}_1 + u = 0, \quad \ddot{y}_2 + 3\dot{y}_2 + 3y_2 - u = 0$$

These test were derived by first identifying the smallest subsets with one redundant equation, e.g., $\{e_1, e_6, e_7\}$, and then eliminating the unknown variables.

Constraints

The model can be extended by adding the constraints

$$c_1 : x_1 \in [0, h_1], \quad c_2 : x_2 \in [0, h_2], \quad c_3 : x_3 \in [0, h_3]$$

to the model.

Trivial Examples

Two simple examples are used to illustrate how the constraints can be used to construct tests.

⋆ Equation e_6 and constraint c_1 give the obvious test

$$y_1 \in [0, h_1]$$

⋆ From e_4 and e_7 it follows that $y_2 = x_1 - x_2$, and using c_1 and c_2 the test

$$y_2 \in [-h_2, h_1]$$

is obtained.

A Not so Trivial Example

The algorithm finds 16 more subsets of equations. For example, a less obvious set that can be used to construct a test is

$$e_2 : \dot{x}_2 = q_1 - q_2, \quad \dot{e}_4 : \dot{q}_1 = \dot{x}_1 - \dot{x}_2, \quad c_2 : x_2 \in [0, h_2]$$
$$e_4 : q_1 = x_1 - x_2, \quad \dot{e}_6 : \dot{y}_1 = \dot{x}_1, \quad c_3 : x_3 \in [0, h_3]$$
$$e_5 : q_2 = x_2 - x_3, \quad \dot{e}_7 : \dot{y}_2 = \dot{q}_1$$
$$e_6 : y_1 = x_1$$

By eliminating x_1, $\dot{x}_1$, $\dot{x}_2$, q_1, q_2, and $\dot{q}_1$ the relation $\dot{y}_1 - \dot{y}_2 - y_1 = x_3 - 2x_2$ is obtained and the corresponding test is

$$\dot{y}_1 - \dot{y}_2 - y_1 \in [-2h_2, h_3]$$

Main Results

The main results of the work can be summarized as:

- Structural characterization of the sought subsets.
- Development of an algorithm that finds all subsets.
- Derivation of the tests.

References

[1] Jan Åslund and Erik Frisk. Structural analysis for fault diagnosis of models with constraints *Proceedings of IFAC Safeprocess'09*, 384–389, 2009, Barcelona, Spain.

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