

# Algorithm for Multiple Fault Isolability Analysis and Computing Test Supports

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## Summary

When designing diagnosis systems for large non-linear dynamic systems, structural methods can be used to find all subsets of equations containing analytical redundancy. Residual generators can be derived from these subsets of equations in the generic case and their fault sensitivities are given by structural properties of the equations. A structural algorithm for computing all subsets of faults such that there exist residual generators sensitive for exactly those subsets of faults has been developed. Since it is complete, the result can be used to characterize multiple fault isolability and to decide which residual generators to implement in the diagnosis system.

## Illustrative Example

Concepts and results will be exemplified by the following illustrative example with unknown signals  $x_i$ , known signals  $u$  and  $y_i$ , and faults  $f_i$ :

$$\begin{aligned} \dot{x}_1 &= -x_1 + u + f_1 & (1) \\ \dot{x}_2 &= x_1 - 2x_2 + x_3 + f_2 & (2) \\ \dot{x}_3 &= x_2 - 3x_3 & (3) \\ y_1 &= x_2 + f_3 & (4) \\ y_2 &= x_2 + f_4 & (5) \\ y_3 &= x_3 + f_5 & (6) \end{aligned}$$

Model Structure:	
	$x_1 \quad x_2 \quad x_3$
(1)	x ← $f_1$
(2)	x x x ← $f_2$
(3)	x x
(4)	x ← $f_3$
(5)	x ← $f_4$
(6)	x ← $f_5$

The models  $\{(4), (5)\}$  and  $\{(3), (5), (6)\}$  are examples of *overdetermined models* with more equations than unknowns. The corresponding residual generators and their fault sensitivities are:

Model	Residual Generator	Test Support
$\{(4), (5)\}$ :	$r_1 = y_1 - y_2 = f_3 - f_4$	$\{f_3, f_4\}$
$\{(3), (5), (6)\}$ :	$r_2 = \dot{y}_3 + 3y_3 - y_2 = \dot{f}_5 + 3f_5 - f_4$	$\{f_4, f_5\}$

## Test Support

It is important which faults each residual responds to and such subset of faults will be called a *test support*. This is a key property for selecting tests such that fault isolation can be achieved.

The model dictates which test supports that are possible for any residual generator. For the example, there is no residual with test support  $\{f_1, f_2\}$  because the corresponding model (1)-(3) has no redundancy. Another example is the set  $\{f_2, f_3, f_4, f_5\}$  which is not a test support because decoupling of  $f_1$  implies decoupling of  $f_2$ .

**Problem Formulation.** Given the structure of a model and in which equation each fault enters find all test supports.

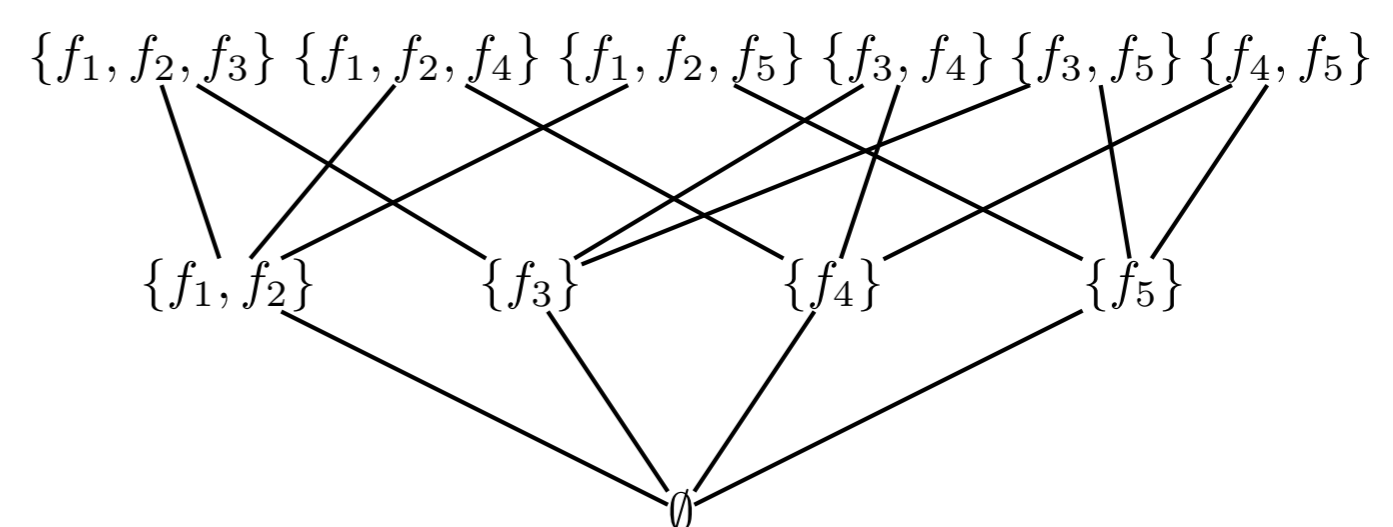
## Fault Isolability Analysis

The complete set of test supports can also be used to characterize multiple fault isolability, i.e., the best possible isolability that can be achieved for any diagnosis systems based on the given model.

Let multiple faults be denoted by their corresponding set of faults, e.g.  $\{f_1, f_2\}$  denotes a double fault. Intuitively, the multiple fault  $F_1$  is isolable from  $F_2$  if there exists a residual generator sensitive to  $F_1$  but not to  $F_2$ . This intuition can be formalized using test supports.

**Theorem.**  $F_1$  is isolable from  $F_2$  if and only if there exists a test support  $F$  such that  $F_1 \cap F \neq \emptyset$  and  $F_2 \cap F = \emptyset$ .

The multiple fault isolability of the example is represented with the following lattice:



Examples of how the lattice should be interpreted:

- $\{f_1\}, \{f_2\}, \{f_1, f_2\}$  is not isolable from each other.
- $\{f_1\}$  is isolable from  $\emptyset$  but not vice versa.
- $\{f_1\}, \{f_3\}$  are isolable from each other.

## Algorithm

The complement sets of the sets of faults given in the lattice are the 11 test supports and this relation is used when constructing the lattice. The algorithm for finding all test supports traverses a spanning tree of the lattice with a depth-first search.

## Truck Engine Study

The algorithm has been applied to a truck engine model with 532 equations, 528 unknowns, and 8 states. We consider faults in 3 actuators and 4 sensors. In previous structural approaches all minimal overdetermined subsets of equations have been computed and in this model there are 1436 such subsets. However, the number of different test supports is 61 and of these only 32 correspond to minimal overdetermined sets. These numbers reflect the computational gain in computing test supports instead of overdetermined models. Furthermore, the test selection problem is simplified by reducing the number of relevant tests from 1436 tests to 61. Finally, the test supports provide a complete multiple fault isolability analysis.