ML Estimation in Mixed Linear/ Nonlinear State-Space Models

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Contributions

- Parameter estimation in mixed linear/nonlinear state-space models.
- An extension of a Rao-Blackwellized particle smoother, capable of handling fully interconnected mixed linear/nonlinear models.

Problem Formulation

The problem is to identify the parameters θ in a mixed linear/nonlinear state space model

$$\begin{aligned} \mathbf{a}_{t+1} &= f_a(\mathbf{a}_t, u_t, \theta) + A_a(\mathbf{a}_t, u_t, \theta)\mathbf{z}_t + w_{a,t}, \\ \mathbf{z}_{t+1} &= f_z(\mathbf{a}_t, u_t, \theta) + A_z(\mathbf{a}_t, u_t, \theta)\mathbf{z}_t + w_{z,t}, \\ y_t &= h(\mathbf{a}_t, u_t, \theta) + C(\mathbf{a}_t, u_t, \theta)\mathbf{z}_t + e_t, \end{aligned} \quad (1)$$

using ***maximum likelihood*** (ML) estimation.

EM Algorithm

Expectation Maximization (EM) is used to compute the ML estimate

$$\widehat{\theta} = \arg\max_{\theta} p_\theta(y_1, \dots, y_N).$$

The **E Step** of the algorithm contains intractable expectations, which are computed approximately using Particle Smoothing (PS).

(E Step): Calculate

$$\mathcal{Q}(\theta, \theta_k) = \mathrm{E}_{\theta_k}\{\log p_\theta(X_N, Y_N) \mid Y_N\}$$

(M Step): Solve

$$\theta_{k+1} = \arg\max_{\theta} \mathcal{Q}(\theta, \theta_k)$$

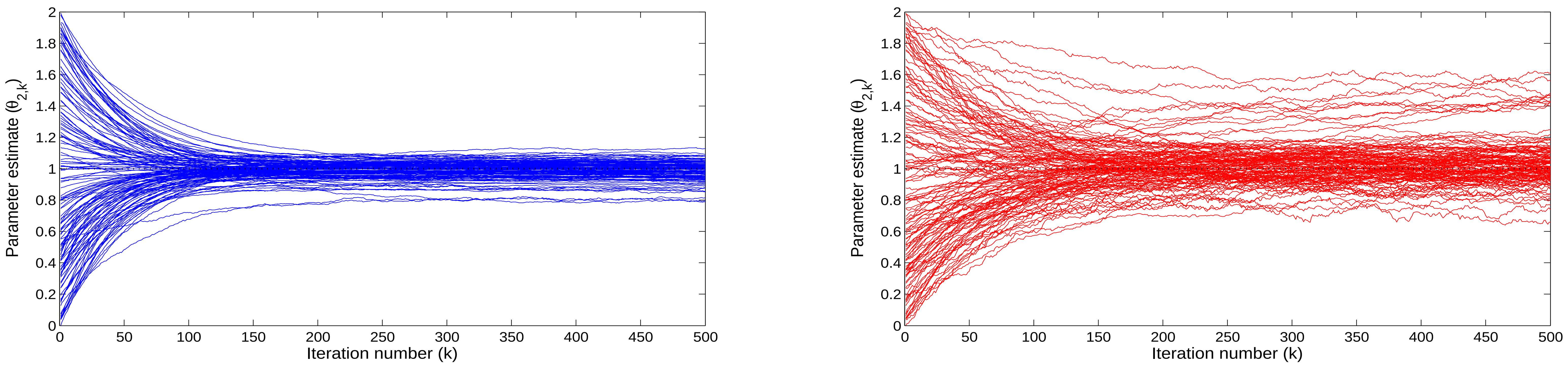

Fig. 1. Parameter estimates vs. iteration number for RBPS-EM (left) and PS-EM (right).

Rao-Blackwellized PS

The Monte Carlo variance of the PS can be reduced by ***exploiting the structure*** in (1), leading to a **Rao-Blackwellized PS** (RBPS).

Previous RBPS only apply to model (1) in special cases ($A_a \equiv 0$). Hence, an existing RBPS has been ***extended to handle the fully interconnected model*** (1) under study.

Experimental Results

The proposed method, based on EM and a new RBPS, is compared with a similar method based on EM and a standard PS. The results from a four-dimensional system ($\dim a_t = 1, \dim z_t = 3$) with one unknown parameter are given in Fig. 1.

Conclusion

Through simulations it has been shown that using a RBPS instead of a PS reduces the variance, not only of the estimated ***states***, but also of the estimated ***parameters***.